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STACKELBERG LEADERSHIP BY A PRICE FOLLOWER

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Abstract

The Cournot-Bertrand model of oligopoly was introduced in the 1970s. It involved some firms being price setters while other firms were output setters. However there remained a question about the identity of the entity which changed the prices of the output setting firms. An alternative formulation of this problem is set out where in an industry there is a price leader and a price follower. But the price follower is a Stackelberg leader. The price follower tries to estimate the reaction function of the price leader through an iterative learning process and incorporate the former into its profit maximisation exercise. Thereby the price follower tries to factor in the consequences of changes in its output on the price it receives. The equilibrium, local stability and analogy of the model with respect to the Cournot-Bertrand model of oligopoly are examined. Such a set-up is compatible with constant returns to scale of the production process of the price follower.

Keywords: Stackelberg Leadership, Price Follower, Constant Returns to Scale, reaction function, Cournot-Bertrand Model, stability.

1. Introduction

Bylka and Komar (1976) introduced the Cournot-Betrand model of oligopoly with some firms setting prices while others set outputs. However Tirole (1988, 209) has pointed out that: "The Cournot model assumes that firms pick quantities rather than prices, and that an auctioneer chooses chooses the price to equate supply and demand". Clearly this is not a satisfactory characterisation of output setting firms. An alternative formulation is to characterise such firms as price followers. But such output setting firms need not be indifferent to the process of price determination. In particular they may seek to know the consequences of changing their output on the price they receive. They could attempt to achieve this goal by trying to estimate the reaction function of the price leader. The

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estimated reaction function of the price leader could then be incorporated into the expression for profits of the price follower. A model which incorporates this feature is the concern of this paper. The rest of the introduction is concerned with a brief survey of the literature on Cournot-Bertrand models of oligopoly, price leadership and Stackelberg leadership.

Debate on the Cournot-Betrand model of oligopoly involved among other things the issue whether it was optimal for firms to be price setters and output setters. Singh and Vives (1984) argued that in a two commodity model if both commodities are substitutes then it is optimal for both firms to set outputs while if both commodities are complements then it is optimal for both firms to set prices. However Sakai, Eguchi, and Ishigaki (1995; Sato and others 1996) demonstrated that Singh and Vives (1984)'s result was not valid if there are three or more commodities. Wang and Ma (2014) demonstrate that in a Cournot-Bertrand oligopoly with two firms and imperfect information where the output setting firm changes output directly in proportion to its marginal profit while the price setting firm has static expectations i.e. it expects the output setting firm to not change its output then the likelihood of stability is higher as compared to a similar setting when there are only price setting or output setting firms.

Several models in the past have examined separately, both Stackelberg equilibrium and price leadership (Varian (1992)). In the following an attempt is made to combine both these concepts. Specifically, a model is set out in which a price follower is a Stackelberg leader. An early model of price leadership is attributable to Forchheimer (1983) which was originally published in German in 1908. In a duopoly setting, price leadership occurs when one firm first sets the price and the other is a price taker (Varian (1992)). If the price leader, sets the price at some level p, then the follower decides its optimal output at a point where the given price (p) equals its marginal cost. This implies that marginal cost of the price follower should be an increasing function of its output (Ono 1982). In other words, in a model of price leadership, the production process of the price follower is characterised by decreasing returns to scale.

Scherer and Ross (1990) has classified price leadership models into three types: dominant, collusive and barometric. The dominant price leadership describe industries having one dominant form with the largest market share with other firms being price followers. The collusive type is one where some principal firms set prices and minor firms follow it. The price is set around the competitive level in the case of barometric price leadership (Ono (1982)).

The Stackelberg model was set out in a book published in German by Heinrich von Stackelberg in 1934 (Von Stackelberg et al. (2010)). A Stackelberg duopoly unfolds sequentially. The Stackelberg follower moves first and maximises its profits assuming that the decision variable of the Stackelberg leaders remains unchanged. The Stackelberg leader observes the optimal choice of the Stackelberg follower and then maximises its profits. Textbook treatments of the Stackelberg model such as Varian (1992), consider a case where the decision variables of both firms is output. Boyer and Moreaux (1987) examine the circumstances that determine the choice of roles by firms i.e. the choice between being a Stackelberg leader and a Stackelberg follower. Boyer and Moreaux (1987) go on to argue that there exist various types of Stackelberg equilibria depending on the relation between the cost and market demand functions of firms.

In Section 2, a model of price leadership by a Stackelberg follower that is an alternative to Cournot-Bertrand oligopoly is set out. Section 2 examines the local stability of the model. Section 3 examines the analogy between the current model and the Cournot-Bertrand model of oligopoly. Some of the comparative static properties of the model are examined in Section 4. Section 5 concludes the paper.

2. A simple duopoly model

Stackelberg leadership by a price follower is examined in a duopoly setting for the sake of simplicity. For the purposes of this paper, a Stackelberg leader is one who incorporates the reaction function of its rival into into its profit maximisation exercise. There are two firms in the industry. Firm 1 is the price leader and Stackelberg follower while firm 2 is the price follower and Stackelberg leader.

The output of firm 1 (q_1) is a decreasing function of its own price (p_1) less the output of firm 2 (q_2).

$$q_1 = a_1 - b_{11}p_1 - q_2 \tag{1}$$

Here a_1 and b_{11} are positive constants.

Firm 1 operates with constant returns to scale i.e. marginal cost (c_1) is constant.[1] The accounting definition of profits of firm 1 is as follows:

$$\pi_1 = p_1q_1 - c_1q_1 \tag{2}$$

The explicit expression for firm 1's profits, after substitution of the expression for the

1 For simplicity it is assumed that fixed costs for both firms are nil.

output of firm 1, can be expressed as follows:

$$\pi_1 = a_1 p_1 - b_{11} p_1^2 - q_2 p_1 - c_1(a_1 - b_{11} p_1 - q_2) \tag{3}$$

Firm 1 chooses its price so as to maximise profits assuming that firm 2 does not change its output:

$$\frac{d\pi_1}{dp_1} = a_1 - 2b_{11} p_1 - q_2 + c_1 b_{11} = 0 \tag{4}$$

The first order condition for maximisation of profits of firm 1 can be solved for the price of firm 1 to yield the reaction function of firm1:

$$p_1 = \frac{a_1 - q_2 + c_1 b_{11}}{2b_{11}} \tag{5}$$

A dynamic formulation of price adjustment by the price leader is one where it increases its price when its marginal profit is positive:

$$\frac{dp_1}{dt} = \alpha \bullet \frac{dq_1}{dp_1} \tag{6}$$

In explicit form, equation (6) may be expressed as:

$$\frac{dp_1}{dt} = \alpha(-2p_1 b_{11} + c_1 b_{11} - q_2 + a_1) \tag{7}$$

Here α is the speed of adjustment of price.

Firm 2 too, operates with constant returns to scale i.e. marginal cost (c_2) is constant. This is unlike traditional models of price leadership as mentioned in Section 1. Further it is assumed that firm 2 has some leeway on the price at which it sells. After all this is not a simple model of price leader and follower but a proposed alternative to a Cournot-Bertrand oligopoly where the output setting firm's characteristics contributes to the determination of the equilibrium both from the side of cost and demand. If the commodities sold by the two firms are differentiated (for instance due to quality differences) it may be assumed that the price of firm 2 is a multiple (m) of the price set by firm 1. Here $m \lesseqgtr 1$. In a first approximation m is assumed to be a constant.

The accounting definition of the profit of firm 2 is as follows:

$$\pi_2 = m p_1^e q_2 - c_2 q_2 \tag{8}$$

Firm 2 is a Stackelberg leader. Therefore it incorporates the reaction function of firm 1 represented by p_1^e into the expression of its profits. Firm 2 needs to estimate the reaction function of firm 1.

Following Bischi, Sbragia, and Szidarovszky (2008) this process of estimation proceeds as follows: firm 2 estimates the the reaction function of firm 1 by assuming it is proportional to the actual reaction function where η is the factor of proportionality. In general η will not be equal to unity. If the actual price is different from the estimated price then it adjusts its scale factor κ. In other words if the actual price exceeds the estimated price then the scale factor of the reaction function of firm 1 (κ) is reduced and vice versa:

$$p_1^e = \frac{(c_1 b_{11} - q_2 + a_1)\eta}{2 b_{11} \kappa} \tag{9}$$

The law of motion of the scale factor of the reaction function of the firm 1 may be expressed as:

$$\frac{d\kappa}{dt} = \gamma(p_1^e - p_1) \tag{10}$$

Here γ is the speed of adjustment of the scale factor of the reaction function of firm 1. The explicit form of the law of motion of the scale factor of the reaction function of the firm 1 is the following:

$$\frac{d\kappa}{dt} = \gamma\left(\frac{(c_1 b_{11} - q_2 + a_1)\eta}{2 b_{11} \kappa} - p_1\right) \tag{11}$$

The explicit expression for firm 2's profits, after substitution of the expression for the estimated reaction function of firm 1, can be expressed as follows:

$$\pi_2 = \frac{q_2 m (c_1 b_{11} - q_2 + a_1)\eta}{2 b_{11} \kappa} - c_2 q_2 \tag{12}$$

The first order condition for maximisation of profits of firm 2 where firm 2 takes into account its estimate of firm 1's reaction function is the following:

$$\frac{d\pi_2}{dq_2} = \frac{(c_1 b_{11} - q_2 + a_1)\eta m}{2 b_{11} \kappa} - \frac{m q_2 \eta}{2 b_{11} \kappa} - c_2 = 0 \tag{13}$$

This process could be expressed dynamically as one where firm 2 increases its output when its marginal profit is positive and vice versa:

$$\frac{dq_2}{dt} = \beta \bullet \frac{d\pi_2}{dq_2} \tag{14}$$

Here β is the speed of adjustment of the output of firm 2. The explicit expression for equation (14) is the following:

$$\frac{dq_2}{dt} = \beta\left(\frac{(c_1 b_{11} - q_2 + a_1)\eta m}{2b_{11}\kappa} - \frac{q_2 \eta m}{2b_{11}\kappa} - c_2\right) \tag{15}$$

The three equations (7), (11) and (15) may be jointly solved to determine the equilibrium of the dynamical system[2]. In such an equilibrium the estimate by firm 2 of the reaction function of firm 1 equals the actual reaction function:

$$p_1^* = \frac{(c_1 b_{11} + a_1)m + 2c_2 b_{11}}{4b_{11}m} \tag{16}$$

$$q_2^* = \frac{(c_1 b_{11} + a_1)m - 2c_2 b_{11}}{2m} \tag{17}$$

$$\kappa^* = \eta \tag{18}$$

Likewise the output of firm 1, by substituting the values of p_1^* and q_2^* from equation (16) and (17) respectively into equation (1) can be expressed as follows:

$$q_1^* = -\frac{(3c_1 b_{11} - a_1)m - 2c_2 b_{11}}{4m} \tag{19}$$

The output of firm 2 and 1 will be positive if the following condition is satisfied:

$$m \succ \frac{2c_2 b_{11}}{c_1 b_{11} + a_1} \tag{20}$$

$$m \succ \frac{2c_2 b_{11}}{3c_1 b_{11} - a_1} \tag{21}$$

If $a_1 \succ 3c_1 b_{11}$ then equation (21) is always satisfied.

Similarly the explicit expressions for the optimal profits of both firms are as follows:

$$\pi_1^* = \frac{(3c_1 b_{11} m - a_1 m - 2c_2 b_{11})^2}{16 b_{11} m^2} = b_{11}\left(q_1^*\right)^2 \tag{22}$$

$$\pi_2^* = \frac{(c_1 b_{11} m + a_1 m - 2c_2 b_{11})^2}{8 b_{11} m} = \frac{m\left(q_2^*\right)^2}{2b_{11}} \tag{23}$$

2 It may be easily verified that the second order conditions for profit maximisation by both firms is readily satisfied.

3. On the local stability of the iterative dynamics

The local stability of the dynamical system consisting of equations (7), (11) and (15) depends on its characteristic equation:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{24}$$

Here λ is the characteristic root. The expressions for the coefficients of the characteristic equation evaluated at the equilibrium are:

$$a_1 = \frac{\beta m}{b_{11}} + \frac{\gamma c_2}{2\eta m} + \frac{\gamma a_1}{4b_{11}\eta} + \frac{\gamma c_1}{4\eta} + 2b_{11}\alpha \tag{25}$$

$$a_2 = \frac{\gamma c_1 \beta m}{4b_{11}\eta} + \frac{\gamma a_1 \beta m}{4{b_{11}}^2\eta} + 2\alpha\beta m + \frac{\gamma c_2 b_{11}\alpha}{\eta m} + \frac{\gamma c_1 b_{11}\alpha}{2\eta} + \frac{\gamma a_1 \alpha}{2\eta} \tag{26}$$

$$a_3 = \frac{\gamma a_1 \alpha\beta m}{2b_{11}\eta} + \frac{\gamma c_1 \alpha\beta m}{2\eta} + \frac{\gamma c_2 \alpha\beta}{\eta} \tag{27}$$

According to Flaschel (2009), the necessary and sufficient conditions for local stability of the concerned dynamical system are:

$$a_1 \succ 0 \tag{28}$$

$$a_3 \succ 0 \tag{29}$$

$$a_1 a_2 - a_3 \succ 0 \tag{30}$$

By inspection it is clear that the two stability conditions represented by equations (28) and (29) are always satisfied. The third local stability condition represented by equation (30) may be set out explicitly as[3]:

$$\frac{\gamma c_1 \beta^2 m^2}{4 b_2^{11} \eta} + \frac{\gamma a_1 \beta^2 m^2}{4 b_3^{11} \eta} + \frac{2\alpha\beta^2 m^2}{b_{11}} + \frac{\gamma a_1 \alpha \beta m}{b_{11}\eta} + \frac{\gamma c_1 \alpha\beta m}{\eta} + \frac{\gamma^2 c_2^1 \beta m}{16 b_{11} \eta^2} + \frac{\gamma^2 a_1 c_1 \beta m}{8 b_2^{11} \eta^2}$$

$$+ \frac{\gamma^2 a_2^1 \beta m}{16 b_3^{11} \eta^2} + 4 b_{11} \alpha^2 \beta m + \frac{2\gamma c_2 b_2^{11} \alpha^2}{\eta m} + \frac{\gamma^2 c_1 c_2 b_{11} \alpha}{2\eta^2 m} + \frac{\gamma^2 a_1 c_2 \alpha}{2\eta^2 m} + \frac{\gamma^2 c_2^2 b_{11} \alpha}{2\eta^2 m^2} + \frac{\gamma c_2 \alpha\beta}{\eta}$$

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3 In equation (31) subscripts denote notation while superscripts represent quadratic terms.

$$+\frac{\gamma\, c_1\, b_2^{11}\, \alpha^2}{\eta}+\frac{\gamma\, a_1\, b_{11}\, \alpha^2}{\eta}+\frac{\gamma^2\, c_1\, c_2\, \beta}{8\, b_{11}\, \eta^2}+\frac{\gamma^2\, a_1\, c_2\, \beta}{8\, b_2^{11}\, \eta^2}+\frac{\gamma^2\, c_2^1\, b_{11}\, \alpha}{8\, \eta^2}+\frac{\gamma^2\, a_2^1\, \alpha}{8\, b_{11}\, \eta^2}+\frac{\gamma^2\, a_1\, c_1\, \alpha}{4\, \eta^2} \quad (31)$$

By inspection it is evident that the local stability condition represented by equation (31) is always satisfied since the relevant expression is always positive. Thus the dynamical system is always locally stable.

4. On the analogy with a Cournot-Bertrand model of oligopoly

In order to facilitate the comparison between the current model and a Cournot-Betrand model of oligopoly a brief recapitulation of a duopoly variant of the latter is undertaken. The demand functions of the two firms (1 denoting the price setter and 2 denoting the output setter) may be represented as[4]:

$$q_1 = a_1 - b_{11}p_1 + p_2 \quad (32)$$

$$q_2 = p1 - b_{22}p_2 \quad (33)$$

If both firms have constant marginal costs and zero fixed costs, the profit expressions of both these firms may be set out as:

$$\pi_1 = -\frac{p_1q_2}{b_{22}} + \frac{c_1q_2}{b_{22}} + \frac{{p_1}^2}{b_{22}} - \frac{c_1p_1}{b_{22}} - {p_1}^2b_{11} + c_1p_1b_{11} + a_1p_1 - a_1c_1 \quad (34)$$

$$\pi_2 = -\frac{{q_2}^2}{b_{22}} + \frac{p_1q_2}{b_{22}} - c_2q_2 \quad (35)$$

The first order conditions of the two firms may be represented as:

$$\frac{d\pi_1}{dp_1} = -\frac{q_2}{b_{22}} + \frac{2p_1}{b_{22}} - \frac{c_1}{b_{22}} - 2p_1b_{11} + c_1b_{11} + a_1 = 0 \quad (36)$$

$$\frac{d\pi_2}{dq_2} = -\frac{2q_2}{b_{22}} + \frac{p_1}{b_{22}} - c_2 = 0 \quad (37)$$

The equilibrium values of p_1 and q_2, obtained by jointly solving equations and may be expressed as:

4 In order to draw a close connection between the two models it has been assumed that the intercept term of the demand function of firm 2 is zero and the sensitivity of each firm's demand with respect to the price of its rival is unity. The latter assumption means that the output of both firms are substitutes.

$$\overline{p_1} = \frac{(2c_1b_{11}+c_2+2a_1)b_{22}-2c_1}{4b_{11}b_{22}-3} \tag{38}$$

$$\overline{q_2} = -\frac{2c_2b_{11}b_{22}^2+(-c_1b_{11}-2c_2-a_1)b_{22}+c_1}{4b_{11}b_{22}-3} \tag{39}$$

Since in equation (38), $\overline{p_1} \succ 0$ even when the marginal costs are zero ($c_1 = 0 = c_2$) it follows that the denominator of equation (38) must be positive:

$$4b_{11}b_{22} - 3 \succ 0 \tag{40}$$

In addition to the inequality set out in equation (40), a sufficient condition that subsumes equation (40) and ensures that the numerators of both equations (38) and (39) are positive is:

$$b_{11}b_{22} - 1 \succ 0 \tag{41}$$

In both models the price of firm 1 and the output of firm 2 are functions of five parameters namely three parameters representing demand/pricing and two parameters representing costs. It may be kept in mind that the marginal revenue of firm 2 in the model of the paper is a decreasing function of its output since it seeks to incorporate its estimate of the reaction function of firm 1, which is a linear decreasing function of the output of firm 2, into its profit maximisation exercise.

An examination of the comparative static properties of both models is now undertaken:

(i) When there is an increase in a_1, then firm 1 increases its price and firm 2 increases its output in both models:

$$\frac{d\overline{p_1}}{da_1} = \frac{2b_{22}}{4b_{11}b_{22}-3} \succ 0 \tag{42}$$

$$\frac{dp_1^*}{da_1} = \frac{1}{4b_{11}} \succ 0 \tag{43}$$

$$\frac{d\overline{q_2}}{da_1} = \frac{b_{22}}{4b_{11}b_{22}-3} \succ 0 \tag{44}$$

$$\frac{dq_2^*}{da_1} = \frac{1}{2} \succ 0 \tag{45}$$

In both models and for both firms, a rise in a_1 will increase their marginal revenue. Since marginal cost is fixed for both firms, a rise in the price of firm 1 and the output of firm 2 respectively are required to reduce their respective marginal revenues to the fixed levels of their respective marginal costs.

(ii) When there is an increase in c_1, then firm 1 increases its price and firm 2 increases its output in both models:

$$\frac{d\overline{p_1}}{dc_1} = \frac{2(b_{11}b_{22}-1)}{4b_{11}b_{22}-3} \succ 0 \quad (46)$$

$$\frac{dp_1^*}{dc_1} = \frac{1}{4} \succ 0 \quad (47)$$

$$\frac{d\overline{q_2}}{dc_1} = \frac{b_{11}b_{22}-1}{4b_{11}b_{22}-3} \succ 0 \quad (48)$$

$$\frac{dq_2^*}{dc_1} = \frac{b_{11}}{2} \succ 0 \quad (49)$$

A rise in the marginal cost of firm 1 in both models requires a rise in its marginal revenue which is brought about through a rise in its price. A rise in the the marginal cost of firm 1 increases the marginal revenue of firm 2 in both models. Since the marginal cost of firm 2 is fixed, a rise in the output of firm 2 is required to ensure that its marginal revenue falls in order to restore equilibrium in both models.

(iii) When there is an increase in c_2, then firm 1 increases its price and firm 2 decreases its output in both models:

$$\frac{d\overline{p_1}}{dc_2} = \frac{b_{22}}{4b_{11}b_{22}-3} \succ 0 \quad (50)$$

$$\frac{dp_1^*}{dc_2} = \frac{1}{2m} \succ 0 \quad (51)$$

$$\frac{d\overline{q_2}}{dc_2} = -\frac{2b_{22}(b_{11}b_{22}-1)}{4b_{11}b_{22}-3} \prec 0 \quad (52)$$

$$\frac{dq_2^*}{dc_2} = -\frac{b_{11}}{m} \prec 0 \quad (53)$$

A rise in the marginal cost of firm 2 requires a rise in its marginal revenue which is brought about through a rise in its output in both models. A rise in the output of firm 2 reduces the marginal revenue of firm 1 in both models. Since the marginal cost of firm 1 is fixed, a rise in the price of firm 1 is required to ensure that its marginal revenue falls in order to restore equilibrium in both models.

Since the assumption about the demand functions in the simplified Cournot-Bertrand model resulted in the output of both commodities being substitutes while this need not be the case in the model of the current paper, a comparison of changes in b/b_{11} and m/b_{22} is not undertaken here. That will require a prior endogenous determination of m.

5. Conclusion

This paper analyses the equilibrium in a setup where the price leader is a Stackelberg follower. Some of the new results include:

(i) Contrary to the analysis of price leadership presented in Ono (1982), the production process of the price follower is characterised by constant returns to scale.

(ii) A certain analogy is possible between the model of this paper and the Cournot-Bertrand model of oligopoly but without a need for an auctioneer to determine the price of the price of the output setting firm.

No claim has been advanced as to the empirical relevance of the model that has been set out since the model itself is very preliminary. The local stability result does not admit the possibility of cycles or complex dynamics. Future work could also examine the stability of an extended model which could incorporate:

(i) An endogenous determination of m which is the mark up (or mark down) that firm 1 applies to the price set by firm 1. Within the framework of the model of this paper one factor that could play a role is the quality of the output of firm 2 vis-à-vis that of firm 1 or in other words the extent of product differentiation[5].

(ii) Many price leaders and price followers including the possibility of an endogenously determined number of price followers.

(iii) Nonlinear demand functions.

(iv) An attempt by the price leader too to exercise Stackelberg leadership.

(v) Joint production by both the price leader and the price follower.

5 Sato and others (1996) refer to the situation in the 1990s of the “Japanese home electronics industry where Matsushita, Sanyo and Sony are three major companies. Matsushita tends to employ sales expansion strategies by means of its powerful distribution channels. In contrast, Sanyo favours a price-cutting strategy, while Sony is keen to sell quality products.”

(vi) Some work on experimental evaluation of the model is ongoing and the results obtained will be reported in due course.

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